Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MFP4

Unit Further Pure 4

Tuesday 18 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The points A, B, C and D have position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ respectively relative to the origin O, where

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$$

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. *(3 marks)*

(b) The points A, B and C lie in the plane Π. Find a Cartesian equation for Π. *(2 marks)*

(c) Find the volume of the parallelepiped defined by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$. *(3 marks)*

QUESTION PART REFERENCE

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2 The system of equations

$$\begin{aligned} 2x - \;\; y - \;\; z &= 3 \\ x + 2y - 3z &= 4 \\ 2x + \;\; y + az &= b \end{aligned}$$

does not have a unique solution.

(a) Show that $a = -3$. *(3 marks)*

(b) Given further that the equations are inconsistent, find the possible values of b. *(2 marks)*

(c) State, with a reason, whether the vectors $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ -3 \\ -3 \end{bmatrix}$ are linearly dependent or linearly independent. *(1 mark)*

QUESTION PART REFERENCE

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3 The determinant Δ is given by

$$\Delta = \begin{vmatrix} x^2 - x & y^2 - y & z^2 - z \\ x & y & z \\ x^2 + y^2 + z^2 & x^2 + y^2 + z^2 & x^2 + y^2 + z^2 \end{vmatrix}$$

where x, y and z are distinct real numbers.

(a) Express Δ as a product of one quadratic factor and three linear factors. *(6 marks)*

(b) Deduce that $\Delta \neq 0$. *(2 marks)*

QUESTION PART REFERENCE

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4 Two planes have equations

$$2x - 2y + z = 24$$

and

$$x + 3y + 4z = 8$$

They meet in a line L.

(a) Find Cartesian equations for the line L. *(5 marks)*

(b) The direction cosines of the line L are given by $\cos\alpha$, $\cos\beta$ and $\cos\gamma$.

(i) Find the exact value of each of the direction cosines. *(2 marks)*

(ii) Show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$. *(3 marks)*

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5 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$$

(a) Show that $\lambda = 2$ is an eigenvalue for $\mathbf{M}$, and find the other two eigenvalues. *(5 marks)*

(b) Find an eigenvector that corresponds to $\lambda = 2$. *(3 marks)*

(c) The matrix $\mathbf{N}$ is given by

$$\mathbf{N} = \begin{bmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{bmatrix}$$

(i) Show that $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector for $\mathbf{N}$, and find the corresponding eigenvalue. *(2 marks)*

(ii) Hence state one eigenvector for the matrix $\mathbf{MN}$, and find the corresponding eigenvalue. *(3 marks)*

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6 The plane transformation T is defined by

$$\mathrm{T} : \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) A shape has an area of 3 square units. Find the area of the shape after being transformed by T. *(2 marks)*

(b) (i) Find the equations of all the invariant lines of T. *(5 marks)*

(ii) State the equation of the line of invariant points of T. *(1 mark)*

QUESTION PART REFERENCE

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7 The 3×3 matrices $\mathbf{A}$ and $\mathbf{B}$ satisfy

$$\mathbf{AB} = \begin{bmatrix} k & 8 & 1 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix}, \text{ where } \mathbf{A} = \begin{bmatrix} k & 6 & 8 \\ 0 & 1 & 2 \\ -3 & 4 & 8 \end{bmatrix}$$

and k is a constant.

(a) Show that $\mathbf{AB}$ is non-singular. *(1 mark)*

(b) Find $(\mathbf{AB})^{-1}$ in terms of k. *(5 marks)*

(c) Find $\mathbf{B}^{-1}$. *(4 marks)*

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8 A line and a plane have equations

$$\frac{x-3}{p} = \frac{y-q}{3} = \frac{z-1}{-1}$$

and $$\mathbf{r} \,.\begin{bmatrix}1\\1\\2\end{bmatrix} = 10$$

respectively, where p and q are constants.

(a) Show that the line is **not** perpendicular to the plane. *(1 mark)*

(b) In the case where the line lies in the plane, find the values of p and q. *(4 marks)*

(c) In the case where the angle, θ, between the line and the plane satisfies $\sin\theta = \dfrac{1}{\sqrt{6}}$, and the line intersects the plane at $z = 2$:

(i) find the value of p; *(5 marks)*

(ii) find the value of q. *(2 marks)*

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END OF QUESTIONS